Sufficient conditions for uniform convergence of random series

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ABSTRACT

Sufficient conditions for the uniform convergence of random series are obtained.

Keywords: Random field distribution, sequence of independent Gaussian random fields, sufficient conditions for uniform convergence of random series.

1. INTRODUCTION

Let $\xi_n(t)$, $n \geq 1$, be a sequence of random fields defined and continuous with probability one on a metric compactum (T, d).

We denote by C (T) the spaces of all continuous functions defined on T with a uniform metric.

For each $\xi_n(t)$ we define on (C (T), F), where F - σ is the algebra of Borel sets in C (T), the measure μ_n n is as follows:

$$\mu_n(A) = P\{\xi_{n(\cdot)} \in A\}$$

for each A ∈ F.

This measure is called the distribution of the random field $\xi_n(t)$.

We say that the distributions μ_n, n ≥ 1, are uniformly dense if, for each ε >0, there exists a compact set K_ε such that

$\mu_n(K_\varepsilon) > 1-\varepsilon$ for all n ≥ 1.

Consider a series of the form

$$S(t) = \sum_{n=1}^{\infty} \xi_n(t), \quad (1)$$

where $\xi_n(t)$, $n \geq 1$ is a sequence of independent Gaussian random fields whose implementations with probability one belongs to C (T).

In [1, 2], it was shown that in matters related to the convergence of random series (1), an important role is played by conditions providing a uniform distribution density $\xi_n(t)$.

In this paper, we give sufficient conditions for the uniform convergence of some random series. These conditions are a generalization of the corresponding results obtained in [3,4].

We denote by $N_d (T, \varepsilon)$ the smallest number of open balls whose radius does not exceed ε covering the compactum T.

Condition A. Assume that for some positive К, $\alpha < \infty$ and sufficiently small ε > 0, $N_d(T,\varepsilon) \leq \varepsilon^{-\alpha}$ holds.

Note that condition A is satisfied when T is compact in the finite-dimensional space Rn [5]. In what follows, we need the following theorem.

Theorem 1. (see [6]). Let $\xi_n(t)$, $n \geq 1$ be a sequence of Gaussian random fields in C(T), $M\xi_n(t)=0$. If condition A is satisfied and
1) there exists $s_0 \in T$ such that

$$\lim_{C \to \infty} \sup_n P\{|\xi_n(s_0)| > c\} = 0$$

2)there exists a non-negative continuous function φ(x) such that

$$\sup_{n} M \mid \xi_n(t) - \xi_n(s) \mid^2 \leq \varphi^2(h)$$

for all t,s ∈S such that d(t,s)<h for some h> 0,

3) φ(x) monotonically increases on a certain interval (0, α) and

$$\int_0^\alpha \frac{\varphi(u)}{u \mid \log u \mid^{1/2}} du < \infty, \quad (2)$$

then the distributions of random fields □n (t) are uniformly dense.

Note that $\varphi^2(u) = \dfrac{1}{\mid \ln \mid u \mid \parallel^{1+\varepsilon}}$ satisfies condition (2). Using Theorems 1, we can prove the

following theorem, which is of independent interest.

Theorem 2. let, $\xi_n(t), \quad n \geq 1$ be a sequence of independent Gaussian random fields in C (T),

Mξ$_n$(t)=0 and series (1) converge with probability one on T. If condition A is satisfied and

$$1) \quad \sup_{d(t,s)<h} M \mid s(t) - s(s) \mid^2 \leq \varphi^2(h);$$

2) φ(x) monotonically increases on a certain interval (0, α);

$$3) \int_0^\infty \frac{\varphi(u)}{u \mid \log u \mid^{1/2}} du < \infty,$$

then series (1) converges uniformly with probability one on T.

Evidence. Let $S_n(t) = \sum_{k=1}^{n} \xi_k(t)$ the partial sum of the series (1). It is known [1] that in order

for series (1) to converge uniformly with probability one, it is necessary and sufficient that the distributions generated by the partial sums Sn (t) be uniformly dense. To do this, we need to verify that condition 1.2 of Theorem 1 holds for Sn (t).

Since series (1) for each t∈T converges with probability one, condition 1 of Theorem 1 is satisfied. We verify condition 2 of Theorem 1.

Put ΔS = S(t) – S(s) и $Z_n(t) = \sum_{k=n+1}^{\infty} \xi_n(t)$. As S(t)=S$_n$(t)+Z$_n$(t)

a S$_n$(t) и Z$_n$(t) independent then

$$M \mid \Delta S(t) \mid^2 = M \mid \Delta S_n(t) + \Delta Z_n(t) \mid^2 =$$

$$= M \mid \Delta S_n(t) \mid^2 + 2M \Delta S_n(t) \cdot M \Delta Z_n(t) + M \mid \Delta Z_n(t) \mid^2,$$

but $M \mid \Delta S_n(t) \mid = 0$, consequently $M \mid \Delta S(t) \mid^2 = M \mid \Delta S_n(t) \mid^2 + M \mid \Delta Z_n(t) \mid^2$

That $M \mid \Delta S_n(t) \mid^2 \leq M \mid \Delta S(t) \mid^2$,

$$\sup_{d(t,s)<h} M \mid \Delta S_n(t) \mid^2 \leq \sup_{d(t,s)<h} M \mid \Delta S(t) \mid^2 \leq \varphi^2(h)$$

or

$$\sup_{d(t,s)<h} M \mid S_n(t) - S_n(s) \mid^2 \leq \varphi^2(h)$$

those. condition 2) of Theorem 1 is satisfied.

The theorem is proved.

Let $\{f_k(t)\}$ be a sequence of continuous functions on T, and ξ_k a sequence of mutually independent Gaussian random variables such that M $\xi_k = 0$. We give a theorem giving conditions for uniform convergence with probability, a unit of series of the form

$$\eta(t) = \sum_{k=1}^{\infty} \xi_k f_k(t)$$

Theorem 3. If

1) $\displaystyle\sup_{d(t,s)\leq h} |f_k(t) - f_k(s)|^2 \leq c_k \varphi^2(h)$;

2) $\displaystyle\sum_{k=1}^{\infty} C_k M \xi_k^2 < +\infty$;

3) $\varphi(x)$ monotonically increases on a certain interval $(0, \alpha)$;

4) $\displaystyle\int_0^{\alpha} \frac{\varphi(u)}{u\,|\log u|^{1/2}}\,du < \infty$,

then the series $\eta(t)$ converges uniformly with probability one on T.

Evidence. It's enough to note that

$$\sup_{d(t,s)\leq h} M\,|\eta(t) - \eta(s)|^2 = \sup_{d(t,s)\leq h} \left| \sum_{k=1}^{\infty} \xi_k f_k(t) - \sum_{k=1}^{\infty} \xi_k f_k(s) \right|^2 \leq$$

$$\leq \sum_{k=1}^{\infty} M\xi_k \sup_{d(t,s)\leq h} [f_k(t) - f_k(s)]^2 \leq \sum_{k=1}^{\infty} C_k M\xi_k^2 \varphi^2(h) = C\varphi^2(h),$$

Where $C = \displaystyle\sum_{k=1}^{\infty} C_k M\xi_k^2$, i.e. condition 1) of Theorem 2 is satisfied.

The theorem is proved.

Note that for random processes Theorem 2 was proved in [7], and in the case when the compactum T in Rn, similar conditions were considered [3,4].

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